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# A Test for Classical Psychospinors †, ‡

by

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**Abstract**

The present paper supplements Oshins' ANPA West 8 paper, "The Search for Classical Psychospinors," by including a specific experimental test proposed by Oshins during the post-session. After a short summary of Oshins' work on *classical* and *quantum* spinor representations in psychology, a brief overview of the history of certain hand motions found in some martial and meditative arts is discused. Shepard's experimental demonstration of the phenomena of mental rotation of internal imagery is described. Georgopoulos' rotating, neuronal "population vector" realization for Shepard's cognitive model is briefly described. Hamilton's realization of SU(2,C) as "turns" is discussed. Georgopoulos' neurophysiological approach to Shepard's cognitive mental rotation model is adapted to Oshins' proposal for *classical* psychospinor representations for self-referential motion. Oshins' "population turn" hypothesis for neurophysiological correlates of *classical* psychospinors is made explicit. Extensive end notes provide technical elaborations upon the content of the paper.

## Section I: Introduction

In 1976 I proposed using *spinor* representations of the Pauli rotation-reflection algebra in psychology (Oshins & McGoveran, 1980; Oshins 1982, 1984a,c,d). This was done in order to reconcile controversy in the psychological literature over schizophrenia, treated as a logical phenomena (Oshins and McGoveran, 1980; Oshins 1982b/1983 rev., 1984a, 1987b, 1989a,b,c(to appear), 1992b, 1994 (to appear); Hilgard, 1989), and as a formal alternative to Brown's (1973) *Laws of Form* approach to self-referential paradoxes (Oshins and McGoveran, 1980; Oshins 1990, 1991a,b, 1993a.).

In Oshins and McGoveran (1980), we also proposed using spinor representations in order to realize Shepard's mental rotations as unitary transformations in Hilbert space [1]. In particular, I was interested in how and why the brain would code the natural "orientation-entanglement relation" of the human arms in terms of quaternions and spinors: If one picks up an object, such as an alarm clock, and rotates it $360^0$ with respect to some reference frame, such as oneself; it winds up where it started. In contrast, if one holds ones hand with the palm up, as if holding a cup of tea on the palm, and rotates it, while keeping the palm up so as not to spill the tea, one finds it necessary to turn the palm around the rest of oneself *twice* ( i.e. $720^0$), once below the elbow and once above it, before one winds-up where one is/was.[2,3,4,5] This is illustrated thus (adapted from Bernstein & Phillips, 1981):

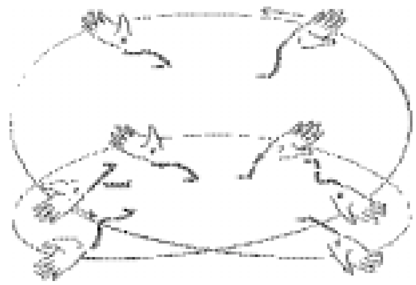


Figure 1: Double Covering of Palms found in Martial Arts Pa Kua Chang and Pencak Silat and in the Philippine folk dance Binasuan ("wine dance").

I have proposed two psychological interpretations for spinors:

(1) as a solid object/impenetrable region of space (Biedenharn & Louck. 1981, Ch. 2, pp. 7-26; Ch. 4, pp. 180-203; and Ch. 7, 7.10, Sect. b., pp. 528-532 ; Oshins, (Oshins 1983c, 1984a,c, 1985, 1986a/87e, 1987b, 1988a, 1993b), instead of rigid objects (Shepard & Chipman, 1970; Shepard & Metzler, 1971; Shepard & Cooper, 1976; Shepard, 1978, 1979, 1984; Carleton & Shepard, 1990a,b). Such *classical spinors* are even-dimensional, *continuous realizations* of the simply-connected, double covering group [SU(2,C)] of the odd-dimensional rotation group [SO(3,R)][6]; and

(2) as quantum dichotomies/bits with ambiguity, obeying Dirac's "principle of linear superposition" of *rays* in projective Hilbert space. This latter lead to my "synaptic spanning" model for "*quantum* parallel processing,"[7] presented as a quantum alternative to McCulloch-Pitts' "synaptic summation" model (McCulloch & Pitts, 1947) and to Pribram's hologram/"synaptic superposition" model (Pribram, 1971). *Quantum spinors* are *dichotomic* [2-valued] realizations [eg. "right/left", "up/down"] of the same symmetry group [SU(2,C)], such that the dichotomy itself can be realized as collineating with any continuously parameterized direction — the vector associated with the dichotomy can have any direction in 3-space. (Oshins 1984a,c, 1985, 1986a/87e, 1987b. [ft.nt.10 of note 10 and note 14], 1988a, 1993b; Oshins, et. al., 1992/89).

As a result of citing a preprint by Yuri Orlov (Oshins & McGoveran, 1980), I was asked to serve as scientific spokesperson for Orlov's related work on "doubt states" and "the wave logic of consciousness" (Orlov, 1981, 1982; Oshins, 1983a), smuggled from the Soviet prison camp (Greenbaum, 1986; Oshins, 1983c, 1984c, 1986b, 1987b). Although Orlov explicitely insisted that his wave logic was not of a quantum nature (Oshins, 1991, ft.nt. 6 and references therein) — i.e. "Our hypothesis is that the experience of doubt is not of a quantum mechanical nature. ..." (Orlov, 1981, p. 88) — it was close enough that I became convinced that if I could show a basis for using quantum physics in psychology, I could provide a tool for physicists in fighting for Orlov's freedom from the prison camp. Since Professor Sidney Drell had urged me to find an empirical basis for my ideas, if I hoped to get the attention of physicists, and since I had already provided a basis to believe in spinor representations in the brain, I turned my attention to possible magnetic effects, ultimately proposing several ideas for experimental inquiries using a SQUID (Superconducting Quantum Interference Device) (Oshins, 1984a,d, 1985, 1987b; Oshins, et. al., 1992/89; Aharanov & Susskind 1967; Bernstein & Phillips 1981).[8]

In 1989, Steve Zins and Myles Hayes independently drew my attention to Geogopoulos's population vector approach to measuring neurophysiological corrolates to Shepard's mental rotations (to be described below). It occurred to me that I might be able to adapt Geogopoulos's technology in my search for *classical* spinor states of the brain. Thus, the origin of this paper.

**Section II: History of Pa Kua Palms** (Jou, 1980; Lu-t'ang, 1983; Miller, D. (ed.). (bi-monthly); Painter, 1981; Smith, 1967; Veith, 1949; Wilhelm, 1967; Ying-arng 1973)

It is said that in the "legendary period" in China the first legendary emperor Fu Hsi divined on the back of a tortoise an archetypical coding for the patterns of nature. Specifically, the story goes that he interpreted cracks on the tortoise as triplets of solid and broken lines. The solid lines stood for yang (masculine, light, active, etc.) and the broken lines stood for yin (feminine, dark, receptive, etc.)[9]. The eight possible triplets of combinations of yin and yang formed the pa kua (8 trigrams). Pairs of trigrams (the inner and outer trigrams) formed the hexigram structure of the *I ching* which is a Chinese "classic" of wisdom and destiny.

The third legendary emperor Huang Ti was the "father of Chinese medicine." He proposed a system of points (tsubo) and channels (merideans) on the human body through which the various "energies" (chi) are supposed to flow. He originated the tradition of acupuncture and the theory of 5 elements (wu hsing). The theory of 5 elements is similar to the childhood game of paper-rock-scissors (hand-fist-fingers). It was supposed to represent the laws of acupunture through so called creative and destructive cycles.[10] In Huang Ti's codification scheme, all of the so-called yin (zang) meridians were on the inside/front of the body and all of the yang (fu) meridians were on the outside/back of the body.[11]

It is said that in the early 1800's a eunich named Tung Hua-chuan came to prominence in the palace of the Ching Emperor. While serving at court, Tung was supposedly carrying his trays in such an agile and skillful manner that he was discovered to be a master of an unknown martial art called pa kua chang (eight trigram palm). Pa kua chang is a so-called "internal" (mind/image/will) art, based upon cultivating one's *intention* and having much in common with such moving meditations as t'ai chi chuan (t'ai chi boxing). One exercise done in this art involves doing a double-covering with both hands coordinated.

In addition, there are two palm positions out of the so-called "8 mother palms" that are essentially the same in form, except for the intention of the change. Specifically, when the hand is held as if embracing someone with the palm facing inward toward the body it is called pao chang (embracing palm). When the hand is held in essentially the same manner except having the intention on the outside of the hand as if going to strike someone with the outside/back surface, it is called liao chang (warding-off palm). So we see that what we are essentially coding is the intent of the hand with respect to the rest of oneself.

Furthermore, there is a series of meditative motions that consist of walking a circle in different poses with the hands in different relative relationships (the so-called 8 "inner palms"). For example, consider the following picture of Sun Lu-Tang, the Pa Kua master and founder of Sun style T'ai Chi [See also pictures, pp. 85-95 in Oshins (1987b)]:

Figure 2: Sun Lu-Tang performing an "inner palm" meditative posture from Pa Kua Chang.

Let us imagine Sun Lu-t'ang's body as being somewhat like a bat or a spreaad out pelt, the palms determining a pair of planes tangent to a right circular cone: As proven in the end notes, a parameterization for a *classical* spinor can be made in terms of the loci of intersection of two planes rotating around the cone (Mercer, 1963; Gaposhkin, personal communications; Zins, personal communications)[12]:

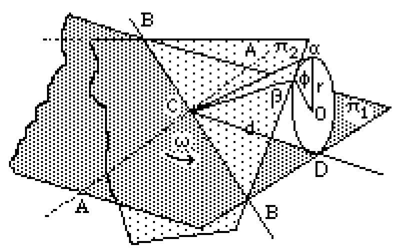



Figure 3. Inner palms as planes circumnavigating a cone as a spinor parameterization.

In the next section I will briefly describe Shepard's pioneering work demonstrating the phenomena of mental rotation of internal imagery and Georgopoulos' extraordinary finding of a cortical "population vector," which is supposed to reflect a monkey's collective, neuronal activity in the motor cortex, corresponding to a cognitive (mental) rotation which is necessary for the monkey to perform a physical task. In the following section I will describe briefly the mathematics of "turns," which provides a realization of the quaternion algebra and thus its spinorial parameterization, suggest that one might adapt Georgopoulos' technology to search for *classical psychospinor*, and provide an explicit test which would demonstrate or reject such an hypothesis.

**Section III: Shepard's mental rotations and Georgopoulos' population vector** (Shepard, 1978, 1979, 1984; Shepard & Cooper, 1976; Shepard & Chipman, 1970; Shepard & Metzler, 1971; Appenzeller, 1989; Georgopoulos, Schwartz, & Kettner, 1986; Geogopoulos, et. al., 1989; Oshins, 1984a,c,d, 1985, 1986a/87e, 1987b, 1988a, 1992a, 1993b; Oshins & McGoveran, 1980; Oshins, et. al., 1989)

**Shepard's Mental Rotations**[13,14]:

Shepard has shown that in mentally comparing differentially oriented, asymmetrical geometric objects, the time required to accurately discriminate whether or not a second object is a mirror image or an equivalent object is linearly proportional to the relative angular orientation, thereby, indicating that rotations of mental representations take place during these comparisons. For example (adapted from Shepard & Metzler), subjects were asked to compare the top block set with the bottom block set in each of the three groups depicted below:

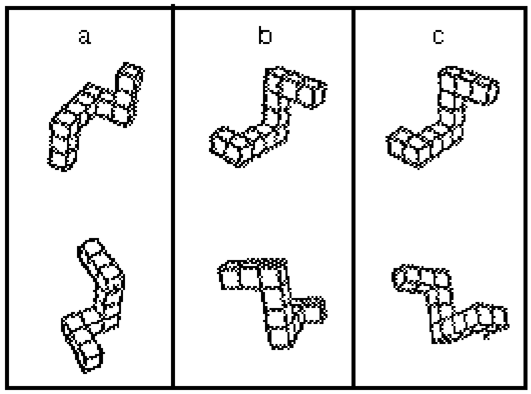



Figure 4 Shepard & Metzler's Mental Rotation Experiment: For figures 4a and 4b, a rotation is all that is necessary to align the objects so that they coincide. For figure 4c this is impossible since one of the pair is the mirror image of the other. The time measured to check for sameness or inversion is linearly proportional to the angle necessary for the rotations.

The reader will be convinced that the first two sets are identical other than in orientation but the last set would require an inversion of one of the pair in order for them to have to be alined. Shepard and his colleagues showed that the amount of time to compare was linearly proportional to the angle through which one would have to rotate (within a plane in 3-space) in order to aline the objects. This frequency was not very sensitive to whether the rotation lay within the picture plane or out of the picture plane, thereby indicating a 3-dimensional aproximate isotropy to the mental rotation. With some practice, the reader will also be able to see the objects apparently rotate in the mental image during the task of making the comparison

**Georgopoulos' Population Vector**[15]:

Georgopoulos and his colleagues showed that they could define a representation of collective activity in the motor cortex of a monkey called a population vector that has a directionality which strongly corrolatable with the direction of motion of a monkey's arm during a conditioned task. Specifically, he found that (1) each neuron has a preferred direction for which its firing rate increased the most; (2) during one movement, neurons with many different preferred directions showed changes in activity; (3) during one movement, cells tuned for directions close to the direction of movement showed the largest increases in firing; and (4) cells tuned to different directions than the direction of movement showed smaller changes such that the frequency of discharge of a particular neuron during movement varied linearly with the cosine of the angle between the neuron's preferred direction and the direction of motion. The population vector is a sum of weighed vectorial contributions of individual cells, where one weighs the preferred direction of the individual cells with the cells' difference in activity level from some offset level[16].

Having found such a directionally tuned, collective, neuronal representation (the population vector), Georgopoulos and colleagues used it to "probe" the cognitive task of a monkey making a mental rotation in order to perform a motor task. After a preparatory signal, the monkey was trained to move a handle in the direction of a dim light, but if the light was bright, the task was to move the handle in a direction $90^o$ counter-clockwise from the dim direction (i.e. perpendicular to it). This was all done in a plane having 8 equally spaced, randomized pairs of directions, Georgopoulos found that the population vector developed before the actual movement and that when the task involved changing the direction of motion, the population vector (representing the collective neuronal activity) rotated uniformly with an approximately constant angular velocity! Thereby, he succeeded in in finding "the first direct visualization of a cognitive process in the brain."

**Section IV: Oshins' Population Turn Hypothesis for Classical Psychospinors** [17] (Oshins, 1992a, 1993b):

### WHAT ARE TURNS[18]

Turns provide a way to represent rotations in 3-space: (1) if you let your thumb (of say your right hand) point in the direction of the axis of rotation, with the other fingers curling around an arc in the direction of rotation; and (2) if you imagine this directed arc to lie on the surface of a (unit) sphere along a great circle and allow its length in degrees (from tale to head) to be 1/2 the angle of rotation, you will observe that you can parameterize all 3-dimensional rotations in terms of these 1/2-angle directed arcs of great circles, or *turns*. [I will not discuss the reason for the 1/2 here. It is in Professor Biedenharn's chapter and devolves around the fact that all translations can be made out of pairs of reflections in parallel planes and all rotations can be made out of pairs of reflections in intersection planes]

One combines turns to realize the rotation *product* by means of *addition* of turns, i.e. one lines up the 1/2-directed arcs head to tail (modulo great circle transport) in a manner that is *similar* to adding vectors heat to tail (modulo parallel transport in order to align them). The result of *adding* two turns gives the turn that corresponds to what would be the result of *multiplying* the two rotations. The fundamental difference between adding turns as opposed to adding vectors is that the order of addition of the two turns in general makes a difference as is the case for rotations (i.e. $T_2 + T_1 \neq T_1 + T_2$ when $R_2 R_1 \neq R_1 R_2$) whereas the result of adding two vectors is order independent (i.e. $V_1 + V_2 = V_2 + V_1$).

### OSHINS' POPULATION TURN HYPOTHESIS (Oshins, 1992a, 1993b):

> *I am predicting that the collective activity pattern which Georgopoulos refers to as a population vector will actually transform as a "population turn" or "population psychospinor". Specifically that: (1) the activity currents will add as turns do to realize the product of the rotation, and (2) that the order of the addition will be relevant as is the case for 3-dimensional rotations. This representation only works for the (more fundamental) 1/2-integral representations (i.e. spinors/turns/quaternions) but also lets one build the vector and tensor representations. The converse does not hold.*

I think that such addition would impart an extraordinary adaptive advantage over multiplication since addition is so much simpler. It would seem to be a natural way for activity to combine and a reasonable, possible evolution of Georgopoulos' efforts. On the other hand it might also not work since it would require the addition to be order dependent. Then again, this property of "noncommutivity" in itself might be valuable in some way.[19]

## Section V: Conclusion

In this paper I have reviewed some pioneering work by Shepard and by Georgopoulos in the mental imagery and neurophysiology of mental rotations, respectively. Reasons have been put forth to suggest that there may be more "fundamental," *classical* spinor brain states that could be measured using Georgopoulos' technology. Furthermore, an experimental hypothesis — that the brain uses spinor representations, not vector representation, in realizing mental rotations — has been proposed which is capable of demonstrating (accepting or rejecting) the hypothesis. Since all translations and rotations can be generated by pairs of spinorial reflection through half angles, in parallel planes or intersecting planes, respectively, the described hypothesis may reveal the method by which humans (and other animals) code sensory perception of Euclidean motions

In addition there is reason to believe that efforts to corrolate flows of cross-sensory modalities, such as found in the motor cortex by Georgopoulos and as predicted in the vision cortex by Carlton, would be very valuable. Similar corrolates with flow of magnetic interference that is anticipated to be found using SQUID helmets would also be possible and would open the door for a nonintrusive ability to monitor brain activity in search for cognitive capacities and capabilities.[20] Such would be invaluable from a policy consideration. Specifically, one might be able to monitor an individual to determine if the individual had the capacity to have consciousness — perhaps through negation permitting synchronizations (Oshins, 1989a,b; Hilgard, 1989) — and thereby choice. This could obviate much of the philisophical bantering about issues such as "criminal insanity" and "date rape." If one can not form the necessary concepts, then one can not be held *responsible* for the consequent actions.

A final comment about the search for psychospinors and the quaternionic arm: Should we have neurophysiological corrolates to the spinorial motion of arms it may, indeed, lead to something very important —– at the level of DNA in psychology, i.e. the building blocks of internal representations of experience. Further investigation should be made into the possibility of coupling the two arms through the waist. [21] This would give $SU(2,C)_L \otimes SU(2,C)_R$. This is close to a representation of the Dirac time-space algebra and thus related to the group SL(2,C) of Finkelstein's relativistic quantum logic. There may well be a developmental coordination faculty that is responsible for the synchronizations (or co-channel) necessary for negation and to compact the linear group into the unitary group of ordinary quantum logic (Oshins, 1984d; 1989a,b and references therein; Hilgard, 1989)

## END NOTES

[1] Specifically, we proposed "the more fundamental, simply connected, covering group SU(2,C) instead of the traditional orthogonal rotation group O(3,R)", specifically, to represent Shepard's work on mental rotations (Shepard & Chipman, 1970; Shepard & Metzler, 1971; Shepard & Cooper, 1976; Shepard, 1978, 1979, 1984) as unitary transformations in Hilbert space (Oshins & McGoveran, 1980, footnote 8; Oshins, et. al., 1984).

Some of the correspondences between orthogonal rotation operators R and their simply-connected unitary covering group elements U are shown below.

$$[\mathbf{r} \rightarrow \mathbf{r}' = \mathbb{R}_u \mathbf{r}] \longleftrightarrow [\mathbf{r}\cdot\sigma_{\mathbf{r}} \rightarrow \mathbf{r}'\cdot\sigma = (\mathbb{R}_u \mathbf{r})\cdot\sigma = \mathbf{r}\cdot(\mathbb{U}_{\mathbf{R}}\sigma\,\mathbb{U}_{\mathbf{R}}^{-1})]$$

$\mathbf{r}$ = position vector , $\mathbb{R}_u$ = orthogonal rotation ,

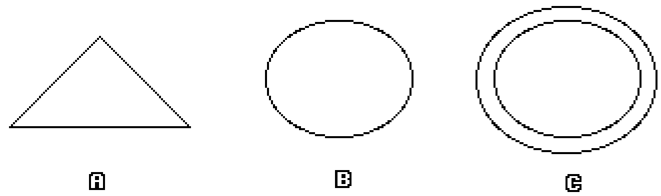
$\sigma$ = Pauli matrix embodyment of spinors , $\mathbb{U}_{\mathbf{R}}$ = unitary rotation ,

$\mathbf{r}\cdot\sigma$ = Cartan matrix representation of vector ,

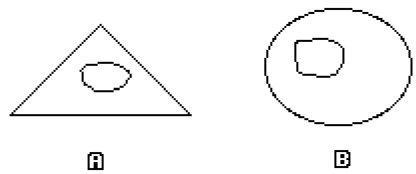
$\pm\,\mathbb{U}_{\mathbf{R}} \overset{2 \rightarrow 1}{\longleftrightarrow} \mathbb{R}_u$ , [2-->1 homomorphism].

The difference between the covering group and the traditional rotation group lies in the global topology, specifically that the covering group is "simply-connected," whereas the traditional rotation group is "doubly-connect" — an issue of "topological homotopy."

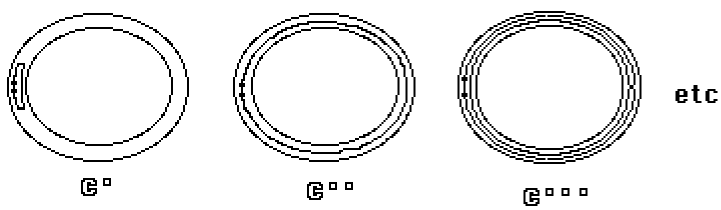
A decade ago, L. Chen (1982) appears to have demonstrated that the perceptual system identifies geometric objects according to their topological homotopy class! If the effects persist, it is a magnificent discovery. Basically, in the type of experiments that he performed, test subjects compared the similarity vs. difference between the following topological structures:



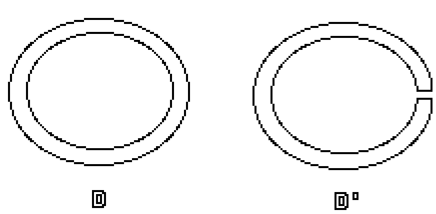

Chen found that test subjects identified the triangle A with the circle B due to their having the same homotopy group, which means that closed paths or loops with the same (arbitrary) base point can be continuously deformed into each other. In this case the fundamental or Poincaré group is the trivial group. This means that all paths are *contractable* and the space is *simply-connected* (like the quaternion covering group of the orthogonal rotation group), and is illustrated below for A and B:




On the other hand, for the ring C, one has more than one possible paths that cannot be shrunk into each other. Indeed, for the ring (annulus) the homotopy/Poincaré group is isomorphic to the group of additive integers Z (likewise, for example, the n-dimensional Torus is isomorphic to the direct sum of n copies of the group of integers). Consider:




In 1984, Steve Zins and I independently suggested the obvious — looking for differences when employing the following two:

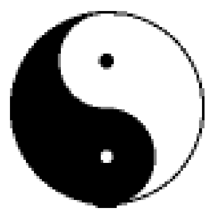



[2] This "double-covering" property of the rotation group was first drawn to my attention by Claude Anderson in 1966. (Mathematicians and physicists refer to this as Dirac's "belt trick" (eg, Kauffman, 1991) I first came to understand the perceptual import of this from Kenneth Cohen in 1980 in his class on Pa Kua Chang at the Academy of Taoist Healing Arts. (Oshins 1983c, 1984a,c, 1985, 1986a/87e, 1987b, 1988a; Painter, B., 1981). (Painter refers to this exercise as "white serpant serves tea." I have been told by several martial

artists that this is an incorrect name. Pa Kua champion Zhang Xian Ming, has told me that the form is called "pang (or ping) wan chung," translated by Pa Kua Sifu Adam Hsu as "leveling hand motion." It refers to ancient practices of leveling the ground with horizontal hand motions when preparing to plant seeds.

This motion is found in other meditative and martial arts, such as the in the Philippine folk dance Binasuan ("wine dance") (Bernstein & Phillips, 1981, p. 122; Oshins, op. cit.; Kauffman, op. cit.) and Pencak Silat (Crista Hansen, personal communication, circa 1987).

[3] If one does the palm up "double covering", with both palms in a symmetric manner with respect to reflection down the medial plane, then one finds that one circumnavigates with the palms a closed ball above the elbows and then with the back of the palms a closed ball under the elbows. In the ancient *Su Wen (Conversations with the Yellow Emperor) (Veith, 1949)*, which is where acupuncture originates, one learns that the ancient Chinese codified the body such that *all yin organs* are on the front side of the body, viz. inside palm, and *all the yang organs* are on the back side of the body, like a turtle, viz. outside palm/back of hand. My claim, and original idea, has been that this is circumnavigating a T'ai Chi (Yin/ Yang) symbol! More recently (Oshins, 1993b) I have suggested that this proximate technique can be used to realize Wing Chun kung-fu's "bong sau/tan sau" movement out of the Kauffman/Oshins "quaternionic arm" discussed and referenced below in end note 5.



End Note Figure 1: T'ai Chi Symbol: If one performs the double-palm covering symmetrically — considering the insides of the palms to be Yin and the outsides of the palms to be Yang — one circumscribes a T'ai Chi symbol. If one approximately does the same movement, assymetrically, with the palms making a *tiny circle*, one effectively is doing Wing Chun's "bong sau/tan sau" movement (Oshins, 1984a, 1986a/87e, 1988a, 1992, 1993b; Oshins, et. al., 1989)

I believe that this may be a way to get mind to code the relative relationship of part of oneself with respect to the rest of oneself (self-referential motion) and can explain the concepts of being "centered"/ "one"/"integrated"/"extended"/"whole" etc. which one strives for in meditation. To this latter idea I am indebted to L.C. Biedenharn for introducing me to finite size/impenetrable region of space spinors (March 10, 1982 letter from L.C. Biedenharn to E. Oshins). Such *classical* spinor representations necessitate an intrinsic 3-dimensionality [i.e. a *solid object*/ impenetrable region of space as opposed to a penetrable *rigid object* (Biedenharn & Louck. 1981, Ch. 2, pp. 7-26; Ch. 4, pp. 180-203; and Ch. 7, 7.10, Sect. b., pp. 528-532 ; Oshins, op. cit. )

[4] One can also do this by fixing the hand frame. One finds that one must walk around oneself *twice* in order to wind up where one began! (Oshins, 1985, 1986a/87e, 1988a). This discovery of mine has been incorrectly credited to Scott Kim on p. 331 of Martin Gardner's recent edition of *The Ambidextrous Universe* (March 18, 1991 letter from Scott Kim to Martin Gardner).

[5] Kauffman and I have also adapted this double covering property in terms of the "quaternionic arm" (Oshins, op. cit.; Oshins, et. al., 1989; Kauffman, 1991; Hart & Kauffman, 1991): Hold the right arm directly in front and perpendicular to the body with the palm up. The right thumb will point to the right.

Rotate the right arm by $180^0$ so that the palm faces down and the thumb points to the left. This realizes $i(180^0/2)$. Next, rotate the arm $180^0$ with the palm continuing to face down, around a verticle axis, until the fingers point towards you and the thumb points to the right. This realizes $j(180^0/2)$. Finally, rotate the arm $180^0$ with the thumb continuing to face right, until the fingers once again point forward. Here we have two choices: If the path is such that the fingers point up during the rotation, one finds that the arm returns to its original position, This realizes $-k(-180^0/2)$ since if the path is such that the fingers point down during the rotation, one finds that the arm does not return to its original position, but to the same position that would obtain by a rotation of $360^0$ around a vertical axis with the palm up during the entire process. This latter choice realizes $k(180^0/2)$ such that $ijk = -1(-360^0/2)$.

[6] SO(3,R) = SU(2,C)/{+1,-1}.

[7] The name "synaptic spanning" comes from the fact that the "span", or the space generated by the states or rays, is the quantum correspondent of logical "or" in the non-distributive, quantum lattice.

[8] For the readers convenience I provide a brief listing of some of these experimental ideas from Oshins (1984a):

(a) Self-Referential Motion: The coherent, chiral superposition of right- and left-handed, information signals and the identification of half-angle representations, when coding the natural "orientation-entanglement relationship" of the human arms, would indicate that the mind uses spinor realizations of mental rotations, which necessitates an intrinsic 3-dimensionality to the coded physical space (solid objects), and simply-connected, global group structures. I have predicted psychological applications of what are known in the physical literature as "Aharanov-Susskind-Bernstein Effects," such as a reversal of spinorial, brain current activity as a consequence of relative self-rotation by $2\pi$.

(b) Symmetry in Nei Chia (Noi Kung): In a manner similar to Bernstein's resolution of translational human movements into Fourier components, I hope to attempt a decomposition of motion of a Chinese "internal" (image-based) martial art (such as Pa Kua Chang or T'ai Chi chuan) into normal modes having rotational symmetries, such as (spinor) spherical harmonics or Bessel functions in order to demonstrate integrated movement and an adaptive economy for certain natural motions.

(c) Projective Ray Representations: The determination of intensity conserving "ray representations", when adding an information state to itself, would provide a quantum alternative to the "hologram hypothesis", a natural interpretation of spatial frequency as "the generator of translations", and an enticing interpretation of "psychic force" and "psychic energy" (viz. Freudian "cathexis") through Heisenberg's equation.

(d) Consciousness and Negatives, Metrics, and Compactness: The capacity for inter-hemispheric synchronization could be responsible for and correlate with a negation function and a metric in cognitive space. (I have proposed a model in which conscious processes would realize a unitary restriction of the 2-dim. special linear group through synchronizing the preparation and the determination of information states, based upon Finkelstein's "relativistic logic". The compact, metric preserving, covering group of the 3-dimensional rotation group, which I have related to Shepard's work, would result.)

(e) Natural Fourier Transforms: The brain might perform a natural Fourier transformation through global phase relations that have a symmetry structure corresponding to the square root of an inversion in a projective unitary gauge space.

(f) Phase-Locked Hallucinations: It may be possible to correlate phase-locked neuronal firing patterns between brain centers during the hallucination of synaesthesic events (eg. seeing a color upon hearing a sound), and of those involving the syncretic fusion of linguistic and sensory modes (e.g. perceiving a foul odor when experiencing a "rotten person").

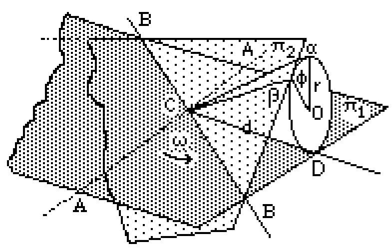
Regarding (8d), I point out that the relation between the Lorentz group SO(3,1;R) and it's simply-connected covering group SL(2;C) is similar to the relation between the orthogonal rotation group and its covering group from end note 6 above, i.e. SO(3,1;R) = SL(2,C)/{+1,-1}. Here, we discover though that whereas the unitary, Lorentz group only has $\infty$ - dimensional representations, the linear, covering group has finite representations. But since they are not unitary, they do not conserve probability as is ordinarily assumed in standard quantum interpretations. Finkelstein's negationless quantum logic and my negationless quantum psychology both employ the special linear group.

[9] The interplay of the yin and the yang formed the T'ai Chi symbol of end note 3, above. Actually, this notion of patterns of change (from yin to yang and from yang to yin) is supposed to have come from the observation that as as the sun passed a mountain the part that was light turned dark and the part that was dark (shadowed) turned light (Veith, 1949; Wilhelm, 1967).

[10] The creative cycle can be identified with $n \to n + 1 mod(5)$ and the destructive cycle can be identified with $n \to n + 2 mod(5)$ (Oshins, 1984d).

[11] See end note 3 above.

[12]

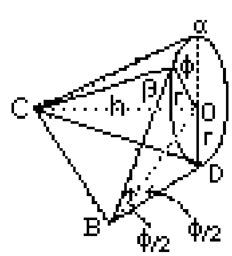



, $\omega = \tan^{-1}(\frac{d}{r}\tan\phi/2)$

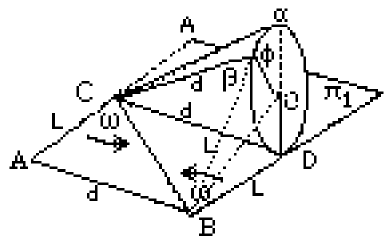
<u>End Note Figure 2: Inner palms as planes circumnavigating a cone</u> as a spinor parameterization.

Let two planes $\pi_1$ and $\pi_2$ lie along the side of a right circular cone such that the lines of tangency of the planes with the cone meet the base of the cone at the points $\alpha$ and D, respectively. The two planes intersect each other along the line AA. As plane $\pi_2$ rotates around the cone, without slippage, the line of tangency to the cone changes its point of intersection with the base of the cone to point $\beta$. Call the new line of intersection of the two planes BB. The base radius $O\alpha$ of the cone travels an angle $\phi$ in the base of the cone when the radius $O\alpha$ travels to the radius $O\beta$, ie $\angle\alpha O\beta = \phi$. Correspondingly, the line of intersection of the two planes $\pi_1$ and $\pi_2$ traces and angle $\omega$ in the plane $\pi_1$, ie $\angle ACB = \omega$.

Let $h$ be the height of the right circular cone and $r$ to be the radius of the right circular cone.



End End Note Figure 2A: Symbol defining picture of: (a) height $h$ of cone; (b) similar right triangles $\Delta OB\beta$ and $\Delta OBD$ which lie in the same plane as the base of the cone.




End End Note Figure 2B: Symbol defining picture of: (a) slant height $d$ of cone; (b) angle $\omega$ measuring the change in the line of intersection of the two planes $\pi_1$ and $\pi_2$; & (c) side $L$ of $\delta OB\beta$ and $\delta OBD$.

Thus, $\angle DBO = \angle\beta BO$. Since radius O is perpendicular to plane $\pi_1$ and radius $O\beta$ is perpendicular to plane $\pi_2$, the sum of $\angle DO\beta + \angle DB\beta = \angle DO\beta + (\angle DBO + \beta BO) = \pi$. Likewise, $\angle DO\beta + aO\beta = \angle DO\beta + \phi = \pi$. Therefore, $\angle DBO = \angle\beta BO = \phi/2$.

Let h be the height, r be the radius, and d be the slant height of the right circular cone. Thus $d^2 = h^2 + r^2$. It follows that the intersection of the two planes provides a 1/2-angle or spinor parameterization:

$$\tan\omega = \frac{d}{L} \quad \tan(\phi/2) = \frac{r}{L} \quad \tan\omega = \frac{d}{r}\tan(\phi/2) \quad \omega = \tan^{-1}(\frac{d}{r}\tan\phi/2)$$

If I consider my *self* to be situated at the locus of intersection of my two palms than *I* should realize such a spinor representation!

[13] Eloise Carlton (personal communication, circa 1983; Carlton, 1988; Carlton & Shepard, 1990a,b; Pribram & Carlton, 1986) has put forth a related, yet fundamentally different, approach toward mental imagery and neurophysiclogical states in visual perception. If I understand her correctly, she has represented cortical activity patterns at a given period of time in terms of the Hilbert space of the electromagnetic field, having a functional form identifiable with Gabor elementary functions (minimum uncertainty packets/Gaussian envelope of plane waves). She then attempts to characterize mental rotations (and other apparent motion) in terms of paths of the unitary action of the Euclidean group acting on this function space. It is hypothesized that this action would correspond to a geodesic path in the group space. The cortical activity paths would thereby be determined by the internal representation path.

This extraordinarily beautiful approach differs from a quantum approach in that the electromagnetic field can *not* carry the unitary representations of a quantum theory as has sometimes been asserted (Pribram & Carlton, 1986; Oshins, 1990, 1991a, & in prep.). Although this model will not work as a quantum model, it may well be a valid and correct classical model.

Furthermore, I do not accept Carlton and Shepard's belief that classical physics is somehow inadequate to represent any motion of a rigid object which can be identified with a path in phase space based upon an alleged difference between what they refer to as "kinetic physics" and "kinematic geometry". It seems to me that they are unnecessarily *defining away* the possibility of "psychic forces" of some sort by insisting that to a physicist an image must follow force-free rectilinear motion. This is interesting since it is standard to go from Galilean invariance (of which the Euclidean group is a subgroup) to Newton's laws to action principles (by imposing covariance) to geometric orbits (via the "action metric"). (Wheeler, 1965, especially the section "geometrical properties of dynamical orbits" in chapter IV, Transformation-theoretic analysis of Newton's equations). Put simply, I believe that there should exist close to an isomorphism between the group geodesic approach of Carlton and Shepard and standard, classical, physical modeling.

In addition, Carlton and Shepard seem to believe that physicists treat the Euclidean group as if the direct product applied to the translation and rotation groups instead of the semi-direct product. This is not the case. In my own approach I identified the Lie algebras of the "generators" of these groups with Schwinger's type I and type II variables, i.e. with momentum and angular momentum. (Schwinger, 1970) That the Euclidean group employs the semi-direct product follows from the fact that the momentum is a vector operator and thereby does not commute with the angular momentum.

Finally, although there are mathematical reasons that make using the covering group of the Euclidean group desireable, I see nothing in the phenomena which they describe that would require using the fundamental spinor representations instead of the ordinary vector representations. But then again, I may be wrong or lacking in understanding of their approach.

In any case, it would be interesting, and possibly very important, to try to identify the flow in the visual cortex suggested by Carlton's representation of Shepard's model for mental rotations, and more general imagery, with the flow in the motor cortex predicted by Georgopoulos' population vector approach. A significant aspect of any such inquiry is that it is difficult (or impossible) to define temporal "zones of *simultaneity"* for different sensory domains of the brain because they have different transduction processes (Ruhnau and Pöppel, 1991). The ability to temporally coordinate such different sensory modalities may be developmentally sensitive and significant (Oshins & McGoveran, 1980, conclusion). It may indeed shed light on the issue of negation and synchronization between preparation and determination of information states which I proposed based upon Finkelstein's relativistic quantum logic (Oshins, 1989a,b; Hilgard, 1989)

[14] I am grateful to Roger Shepard and Eloise Carlton for providing me with access to their work before publication, for trying to help me to understand their approaches, and for comments helpful to my own direction. I am also indebted to Pierre Noyes for his direct help and for his relentless efforts to help us to communicate and understand our varying appraoches.

[15] I would especially like to thank David Adelson and Mike Williams for helping me to understand Georgopoulos' experiment.

[16] This end note sumarizes Georgopolous' computation of a population vector as described in the text:

**<u>Georgopolous' Population Vector Computation:</u>** (3-dim. direction; 282 neurons (41 of which are non-directional) $\therefore$ 224 fit model).

**<u>1. consider a single directionally tuned neuron:</u>**

It is observed that single neurons are *broadly tuned*, i.e. the firing activity changes substantially with movement in *any direction.*

$\vec{M}$ = *movement vector* of unit length defined by directional cosines.

$d(\vec{M})$ = *frequency of discharge of a particular neuron* during motion in the direction of the movement vector $\vec{M}$.

**2. Neuronal ensembles:**

$\vec{C}$ = *preferred movement vector* for which individual cells activity is highest

$\Theta_{cm} = \angle$ between direction of preferred movement vector $\vec{C} =$ and actual movement vector $\vec{M} =$

b is a coefficient that varies from neuron to neuron.

$$d(\vec{M}) = b + kcos\theta_{cm}$$

Single neurons are broadly tuned, i.e. the activity changes substantially with movement in any direction. Thus the movement is not coded by individual cells that respond to movement in only one direction. Instead, we assume that movement is coded by means of a **neuronal ensemble.**

**Three assumptions:**

(1) Each cell indexed by i makes a vectorial contribution along a preferred direction $\vec{C}_i$.

(2) The magnitude of the vectorial contribution of the $i^{\underline{th}}$ cell, i.e. the length (amount) of the vectorial contribution, $w_i$, is a function of the movement direction $w_i = w_i(\vec{M})$. The vectorial contribution $w_i(\vec{M})$ to the ensemble vector is a measure of the change in the $i^{\underline{th}}$ cells activity level from an offset level $b_i$ which represents a baseline activity for the $i^{\underline{th}}$ cell. Specifically,

$$w_i(\vec{M}) = d_i(\vec{M}) - b_i$$

where $d_i(\vec{M})$ is the discharge frequency of $i^{\underline{th}}$ cell for movement in the $\vec{M}$ direction.

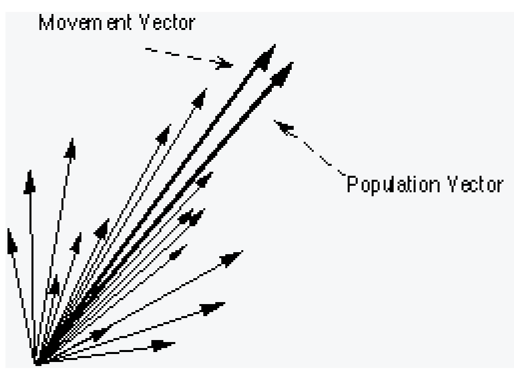
Assumptions 1 & 2 imply that the weighed vectorial contribution of the $i^{\underline{th}}$ cell is:

$\vec{N}_i(\vec{M}) = w_i(\vec{M})\vec{C}_i$

$\vec{N}_i(\vec{M})$ points in the preferred direction $\vec{C}_i$ of the $i^{\underline{th}}$ cell, if $w_i(\vec{M}) > 0$, and points in the opposite direction, if $w_i(\vec{M}) < 0$.

(3) The **Neuronal Population Vector** $\vec{P}_i(\vec{M})$ — which represents a *sharply-tuned, collective directionality* of the 224 broadly- and directionaly-tuned, individual neurons — corresponding to movement in direction $\vec{M}$ results from a *weighed vectorial sum* of the contribution vectors $\vec{N}_i(\vec{M})$ of the individual neuronal cells (See End Note Figure 3, below):

$\vec{P}_i(\vec{M}) = \sum_{i=1}^{224} \vec{N}_i(\vec{M})$




End Note Figure 3: Georgopolous' narrow-band, directional Population Vector (collective activity pattern of 224 wide-band, directional, motor neurons in monkey) aligns up with Movement Vector representing direction of motion: A depiction of the clustering of "jets" of individual neuronal activity corresponding to movement in a direction $\vec{M}$. The weighed vectorial sum of contributions of these individual neurons results in a collective population vector $\vec{P}_i(\vec{M})$ that approximates the movement direction. The movement direction $\vec{M}$ is usually found to lie within a a 95% confidence interval of a cone around the collective neuronal population vector $\vec{P}_i(\vec{M})$ .. (Adapted from Georgopoulos, Schwartz, & Kettner, 1986, Neuronal Population Coding of Movement Direction, *Science*, 26 September 1986, 233: 1416-1419.)

The movement direction $\vec{M}$ is usually found to lie within a a 95% confidence interval of a cone around the collective neuronal population vector $\vec{P}_i(\vec{M})$. It appears from preliminary investigation that the results are insensitive to the origin of motion. (Ibid., ft. nt. 15) [This might indicate that the neuronal population vector is coded in "homogeneous coordinates" and may have a simple projective representation.] It is also found that the direction of the population vector $\vec{P}_i(\vec{M})$ for a particular movement vector $\vec{M}$ was basically insensitive to recalculation using a random sample of 224 neurons drawn with replacement from the original population of neurons or when the number of contributing cells was randomly reduced (Ibid., ft. nts. 11 & 12). Thereby, the neuronal population vector $\vec{P}_i(\vec{M})$ appears to be a robust measure of the direction of movement $\vec{M}$.

In the experiments involving the monkey performing a cognitive task of mental rotation, the following additional observations are pertinent:

(A) All of the experiments have taken place in only one plane of motion — the frontal plane to the monkey (October 31, 1991 letter from Georgopoulos to Oshins). Thus, there is no indication yet that the activity patterns will properly simulate the 3-dimensional rotation group. Nevertheless, it is quite likely that the rotations have approximately a 3-dimensional symmetry as is approximately true in the original mental rotation studies by Shepard and colleagues.

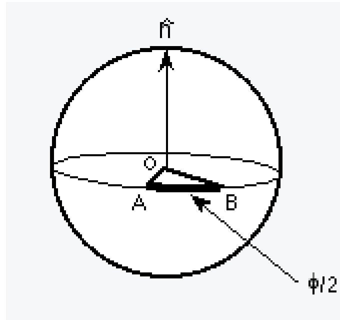
(B) Effects of *coupling both hands* as suggested by Oshins (examples in this context are: Oshins' Aug. 28, 1989 memo to W.A. Little, Oshins' Dec. 31, 1990 letter to L.C. Biedenharn, and Oshins' July 8, 1991 letter to A.P. Georgopoulos) have not yet been attempted (October 31, 1991 letter from Georgopoulos to Oshins).

[17] I am grateful to Larry Biedenharn, Peter Gaposhkin, and Lou Kauffman for helping me to understand some of the relevent mathematics. I have also benefitted greatly from conversations with David Adelson, Fred Young, and Steve Zins on these issues.

[18] This end note sumarizes some of the properties of Hamilton's Turns (largely adapted from Biedenharn and Louck (1981):

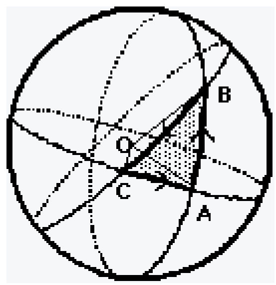
Definition: A *turn* $T_{\underset{\to}{AB}} = \underset{\to}{AB}$

is "half the directed arc of the rotation parametrized as an ordered pair of points on the surface of a unit sphere, modulo great circle transport." [The under-arrow $\underset{\to}{AB}$ is used to reflect the fact that the turn is ordered and directed but is *not a vector* (see Biedenharn and Louck, 1981, Ch. 4, "The theory of turns adapted from Hamilton") which should be clear if only because addition of turns is noncommutative.] The normal $\hat{n}$ to the great circle of the directed arc points in the direction of the axis of rotation using a right handed rule to determine the sense of the rotation. End Note Figures 4A and 4B below illustrate the geometric realization of a turn and the noncommutivity of the addition of turns.




End Note Figure 4A: Geometry of *turn* $T_{AB} = AB$ :

(a) directed arc $\underrightarrow{AB}$ of length $\phi/2$ realizes a rotation by $\phi$; (b) normal $\hat{n}$ points in direction of the axis of rotation.




End Note Figure 4B: Addition of turns $\underrightarrow{AB} + \underrightarrow{BC}$: Turns add head to tail like vectors but the modulo great circle transport. Thus, turns $\underrightarrow{AB} + \underrightarrow{BC}$ = turn $\underrightarrow{CA}$ as described in the text.

Properties of Turns (Biedenharn & Louck, 1981, Sect. 4.2):

1. Two turns are *equivalent* if they can be superimposed by displacing either one along the great circle containing it, i.e. "modulo great circle transport". [This corresponds to *parallel transport equivalence* for the addition of *vectors*].

2. Two turns defined by any pair of diametrically opposite points are equivalent. Let $T_\pi$ denotes this equivalence class. Since $T_\pi$ *commutes* with all turns, it represents a *scalar turn.*

3. Turns form a *group* under addition: In analogy to vectors, the *sum of two turns* is defined by addition head to tail modulo great circle transport . This addition is *noncommutative*, but *associative.* The turn of zero length $T_0(\equiv T_{2k\pi}, k = 0, 1, 2...)$ realizes the *identity* operation (It has *no* length, nor direction $\hat{n}$, nor sense). The *inverse* to a turn $T^{-1}(T_{AB}^{-1}) = -T(+T_{BA})$, i.e. the turn with the same great circle and length, but opposite sense. This group of *Hamilton's turns* is *isomorphic* to SU(2,C) as 3-parameter objects giving a "finite size spinor" as "impenetrable object" in 3-space instead of a 2-parameter object such as Cartan's "point spinor." (Biedenharn and Louck, 1981; Sects. 2.3-2.4 & 2.7, note 2; and pp. 186 & 191)

4. Turns have two involutions:

(A) *inverse turn* $T^{-1}$, i.e. $(T^{-1})^{-1} = -(-T) = T$.

(B) *non-identity, scalar turn* $T_\pi$ associated with a pair of diametrical points — having length p, but *no* direction $\hat{n}$, nor sense — provides a second involution defined as $T^c \equiv T + T_\pi = T_\pi + T$, i.e. $(T^c)^c = (T + T_\pi) + T_\pi = T + T_{2\pi} = T + T_0 = T$.

(C) If one identifies the two equivalence classes of scalar turns $T_0 \equiv T_\pi$, one effectively factors the group SU(2,C) into the proper, orthogonal group SO(3,R), i.e. $Z_2 = \{T_0, T_\pi\}$ and $SU(2,C)/Z_2 = SO(3,R)$.

5. <u>Turns of length $\pi/2(T_{\pi/2})$ have special properties</u>:

(A) Denote a generic turn of length $\pi/2$ by E, i.e. $E \equiv T_{\pi/2}$. It follows that $E^c = E^{-1} = E$ or equivalently that $E + E = T_\pi$.

(B) <u>Theorem</u> (Biedenharn and Louck, 1981, p. 187): An arbitrary turn can always be decomposed (non-uniquely) into a sum of two turns of length $\pi/2$, i.e. $\forall T, \exists E$ and $E'$ such that $T = E' + E$. (Proof: Trivially, let $E' = T_{\pi/2}$ from the tail of T to the intersection of the normal to its plane with the unit sphere and let $E = T_{\pi/2}$ from this point to the head of T.)

(C) <u>Theorem</u> (Biedenharn and Louck, 1981, pp.189-190): If $\hat{e}$ and $\hat{n}$ denote unit vectors from origin perpendicular to planes defined by turns E and T where E is an arbitrary turn of length $\pi/2$ and T is an arbitrary turn, then $\hat{e} \bullet \hat{n} = \cos \parallel T - E \parallel / \sin \parallel T \parallel$, where $\parallel T \parallel$ denotes the length of turn T.

(D) <u>Hamilton's Triplet of "Right Versors"</u> (Ibid.; Hamilton, W.R. (1866/1959): Hamilton's triplet of right versors can be put into a 1-1 correspondence with his quaternions or "quotients of vectors."

[19] Some additional information that would be useful: (1) It would be useful to know the relationship between the left hands activity flows and the activity flows due to the right hand, symmetrically and antisymmetrically, both combined and independently. (2) What might be the relationship between the population vector and spatial frequency. [Spatial frequency data probably relates to translational degrees of freedom (as opposed to rotational) and may be able to be accommodated with the population vector approach as a (the?) way to realize Euclidean motions (translations and rotations)] (cf. also Carlton and Shepard); (3) the population vector uses "homogeneous coordinates" which means that the cell activities all add using the same origin. The cover photo from the original population vector article (<u>Science</u>, 9/26/86) seems to indicate that the population vector is isotropic, i.e. for a similar movement in two different directions the corresponding activities reflect only the movement directions and not an intrinsic directionality in the population vector space. It would be important to know if this is so since state vectors are homogeneous; and (4) There is some data in the psychological literature about unitary vs. analyzable representations of data, where the former uses a Euclidean metric (i.e. which squares the distances, then adds them, then takes the resultant square root) and the latter uses a city-block metric (i.e. which adds the absolute value of the components). Perhaps one might fortuitously use a different metric structure in computing the population vector collective activity? If one wanted to add the activities of both hands, would one add them all up or would one find separate population vectors for each hand and then add them up? These would not be equivalent procedures.

[20] It would also be interesting to see if one could corrolate Georgopolous' population vector with the anticipated magnetic field interference in the brain under the act of mental rotation. Professor W. Fairbank told me in 1984 that there is a plan to make a helmet coupling 30 or so SQUIDs that might be able to find such correlations (See also, "Thinking Cap: Superconducting SQUIDs peer into minds — and hearts," *Scientific American*, March 1991, p. 112).

[21] There is a very famous Chen style t'ai chi chuan exercise called chan-ssu chin (silk-reeling exercise) in which the waist is used in a rotational fashion as the leader and coordinator of the arm motions (Jou, 1980)

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